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On the Capacity and Residual Charge of Dielectrics as Affected by Temperature and Time

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IV. *On the Capacity and Residual Charge of Dielectrics as Affected by Temperature and Time.*

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BEFORE describing the experiments* forming the principal subject of this communication, and their results, it may be convenient to shortly state the laws of residual charge.

Let x_t be the potential at any time t of a condenser, *e.g.*, a glass flask, let y_t be the time integral of current through the flask up to time t, or, in other words, let y_t be the electric displacement, including therein electric displacement due to ordinary conduction. If the potential be applied for a short time ω, let the displacement at time t, after time ω has elapsed from the application of force $x_{t-\omega}$ be $x_{t-\omega}\,\psi(\omega)\,d\omega$; this assumes that the effects produced are proportional to the forces producing them; that is, that we may add the effects of simultaneously-applied electromotive forces. Generalise this to the extent of assuming that we may add the effects of successively-applied electromotive forces, then $y_t = \int_0^\infty x_{t-\omega}\,\psi(\omega)\,d\omega$. This is nothing else than a slight generalisation of OHM's Law, and of the law that the charge of a condenser is proportional to its potential. Experiments were tried some years ago for the purpose of supporting this law of superposition as regards capacity. It was shown that the electrostatic capacity of light flint glass remained constant up to 5,000 volts per millimetre ('Phil. Trans.,' 1881, Part II., p. 365). The consequences of deviation from proportionality were considered ('Proc. Roy. Soc.,' 1886, vol. 41, p. 453), and it was shown that, if the law held, the capacity as determined by the method of attractions was equal to that determined by the method of condensers; this is known to be the case with one or two doubtful exceptions (*ibid.*, p. 458). Rough experiments have been made to show that residual charge is proportional to potential; they indicate that it is ('Phil. Trans.,' vol. 167, Part II.). The integral $y_t = \int_0^\infty x_{t-\omega}\,\psi(\omega)\,d\omega$ includes in itself ordinary conduction, residual

* These experiments were commenced in the summer of 1894, and we have to thank Messrs. C. J. EVANS and R. E. SHAWCROSS for valuable assistance rendered during the period of their Demonstratorship in the Siemens Laboratory, King's College, London.

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charge and capacity. Suppose that from $t = 0$ to $t = t$, $x_t = \mathrm{X}$, and before that time $x_t = 0$, then $y_t = \mathrm{X}\int_0^t \psi(\omega)\, d\omega$, and $\frac{dy_t}{dt} = \psi(t)$; thus $\psi(t)$ is the conductivity after electrification for time t. It has of course been long known that in stating the conductivity or resistance of the di-electric of a cable, it is necessary to state the time during which it has been electrified; hence $\psi(t)$ is for many insulators not constant. $\psi(\infty)$ may perhaps be defined to be the true conductivity of the condenser, but at all events we have $\psi(t)$ as the expression of the reciprocal of a resistance measurable, if we please, in the reciprocal of ohms. For convenience we now separate $\psi(\infty) = \beta$ from $\psi(\omega)$ and write for $\psi(\omega)$, $\psi(\omega) + \beta$. If we were asked to define the capacity of our condenser we should probably say: "suppose the condenser be charged to potential X for a considerable time and then be short-circuited, let Y be the total quantity of electricity which comes out of it, then Y/X is the capacity." If T be the time of charging $y_t = \mathrm{X}\int_0^{\mathrm{T}} \{\psi(\omega) + \beta\}\, d\omega$ at the moment of short circuiting; $y_t = \mathrm{X}\int_t^{\mathrm{T}+t} \{\psi(\omega) + \beta\}\, d\omega$ after time t of discharge. The amount which comes out of the condenser is the difference of these, or $\mathrm{Y} = \mathrm{X}\left\{\int_0^{\mathrm{T}} \psi(\omega) + \beta\, d\omega - \int_t^{\mathrm{T}+t} \psi(\omega) + \beta\, d\omega\right\}$; if t be infinite $\psi(t) = 0$, and $\mathrm{Y} = \mathrm{X}\int_0^{\mathrm{T}} \psi(\omega)\, d\omega$; or we now have capacity expressed as an integral of $\psi(\omega)$ and measurable in micro-farads, and it appears that the capacity is a function of the time of charge increasing as the time increases. Experiments have been made for testing this point in the case of light flint glass, showing that the capacity was the same for 1/20000 second and for ordinary durations of time ('Phil. Trans.,' 1881, p. 356), doubtless because $\int_{1/20000}^{\infty} \psi(\omega)\, d\omega$ is small compared with $\int_0^{1/20000} \psi(\omega) d\omega$. Now $\int_0^t \psi(\omega)\, d\omega$, when t is indefinitely diminished, may be zero, have a finite value, or be infinite; in fact it has a finite value. The value of $\psi(\omega)$ when ω is extremely small can hardly be observed; but $\int_0^t \psi(\omega)\, d\omega$, when t is small, can be observed. It is therefore convenient to treat that part of the expression separately, even though we may conceive it to be quite continuous with the other parts of the expression. $\int_0^t \psi(\omega)\, d\omega$, when t is less than the shortest time at which we can make observations of $\psi(\omega)$, is the instantaneous capacity of the condenser. Call it K and suppose the form of ψ to be so modified that for all observed times it has the observed values, but so that $\int_0^t \psi(\omega)\, d\omega = 0$, when t is small enough.

Then $y_t = \mathrm{K}x_t + \int_0^{\infty} x_{t-\omega}\, \{\psi(\omega) + \beta\}\, d\omega$. Here the first term represents capacity, the second residual charge, the third conductivity, separated for convenience, though really all parts of a continuous magnitude. Suppose now our condenser be submitted to a periodically varying electromotive force, that

$$x_t = \mathrm{A} \cos pt,$$

then

$$y_t = \mathrm{A}\left\{\mathrm{K} \cos pt + \int_0^\infty \cos p(t-\omega)\,[\psi(\omega)+\beta]\,d\omega\right\}$$

$$= \mathrm{A}\left\{\mathrm{K} \cos pt + \cos pt \int_0^\infty \cos p\omega\psi(\omega)\,d\omega + \sin pt \int_0^\infty \sin p\omega\psi(\omega)\,d\omega\right\}.$$

The effect of residual charge is to add to the capacity K the term $\int_0^\infty \cos p\omega\psi(\omega)\,d\omega$, whilst the term $\sin pt \int_0^\infty \sin p\omega\psi\omega\,d\omega$ will have the effect of conductivity as regards the phases of the currents into the flask. Thus the nature of the effect will depend upon the form of the function $\psi(\omega)$. An idea may be obtained by assuming a form for $\psi(\omega)$, say $\psi(\omega) = \frac{\mathrm{C}}{t^m}$, where m is a proper fraction. This is a fair approximation to the truth. Then

$$\int_0^\infty \cos p\omega\psi(\omega)\,d\omega = \Gamma(1-m)\cos(1-m)\,\pi/2/p^{1-m},$$

$$\int_0^\infty \sin p\omega\psi(\omega)\,d\omega = \Gamma(1-m)\sin(1-m)\,\pi/2/p^{1-m}.$$

If m is near to unity, capacity is almost entirely affected; otherwise the effect is divided between the two, and dissipation of energy will occur. It is interesting to consider what sort of conductivity a good insulator such as light flint glass, according to this view of capacity, residual charge, and conduction, would have at ordinary temperatures if we could measure its conductivity after very short times of electrification; if, in fact, we could extend the practice used for telegraph cables and specify that the test of insulation should be made after the one hundred millionth of a second instead of after one minute, as is usual for cables. The capacity of light flint measured with alternating currents with a frequency of two millions a second is practically the same as when measured in the ordinary way; that is, its capacity will be 6·7. Its index of refraction is 1·57 or $\mu^2 = 2{\cdot}46$, or, say, 2·5. We have then to account for 4·2 in a certain short time. The current is an alternating current, and we may assume as an approximation that it will be the residual charge which comes out in one-sixth of the period which produces this effect on the capacity; therefore $\int_0^{1/12\times10^6} \psi(\omega)\,d\omega = \frac{4\cdot2}{6\cdot7} \times$ capacity of the flask as ordinarily measured. The capacity of a fairly thin flask may be taken to be 1/1,000 microfarad to 2/1,000 microfarad; hence we may take $\int_0^{1/12\times10^6} \psi(\omega)\,d\omega$ to be 10^{-9} farad; if $\psi(\omega)$ were constant during this time its value must be $12 \times 10^6 \times 10^{-9} = \frac{1}{80}$ ohms^{-1} about. The value of $\psi(\omega)$

is far from constant, and hence the apparent resistance of that extraordinarily high insulator, a flint-glass flask, must be, for very short times, but still for times enormously large compared with the period of light waves, much less than 80 ohms.

[Added 11th March, 1897.—Somewhat similar considerations are applicable to conduction by metals. MAXWELL pointed out that the transparency of gold was much greater than would be inferred from its conductivity measured in the ordinary way. To put the same thing another way—the conductivity of gold as inferred from its transparency is much less than as measured electrically with ordinary times. Or the conductivity of gold increases after the application of electromotive force. Suppose then we have a current in gold caused by an electromotive force which is increasing, the current will be less than it would be if the electromotive force were constant, by an amount approximately proportional to the rate of increase. If u be the current, ξ the electromotive force, $u = \alpha\xi - \beta\dot{\xi}$ where α is the conductivity as ordinarily measured. This gives us the equation of light transmission $\alpha\dot{\xi} - \beta\ddot{\xi} = \frac{d^2\xi}{dx^2}$ assuming that we have no capacity in the gold.

Professor J. J. THOMSON gives as a result of some experiments by DRUDE that the capacity of all metals is negative. This conclusion is just what we should expect, if we assume, as MAXWELL has shown, that the conductivity of metals increases with the time during which the electromotive force is applied.]

The experiments herein described are addressed to ascertaining the effect of temperature, first on residual charge as ordinarily known, second on capacity as ordinarily known, third to examining more closely how determinations of capacity are affected by residual charge, fourth to tracing the way in which the properties of insulators can continuously change to those of an electrolyte as ordinarily known. The bodies principally examined are soda-lime glass, as this substance exhibits interesting properties at a low temperature, and ice, as it is known that the capacity of ice for such times as one-tenth of a second is about 80, and for times of one-millionth of a second of the order of 3 or less.

RESIDUAL CHARGE AS AFFECTED BY TEMPERATURE.

Experiments on this subject have been made by one of us which showed that residual charge in glass increases with temperature up to a certain temperature, but that the results became then uncertain owing to the conductivity of the glass increasing. These experiments were made with an electrometer, the charge set free in the flask being measured by the rate of rise of potential on insulation. We now replace the electrometer by a delicate galvanometer and measure the current directly without sensible rise of potential.

Fig. 1 gives a diagram of connections. The glass to be experimented upon is blown into a thin flask F, with thick glass in the neck to diminish the effect of charge creeping above the level of the acid, and is filled with sulphuric acid to the shoulder; it is then placed in sulphuric acid in a glass beaker, which forms the inner lining of a copper vessel consisting of two concentric tubes between which oil

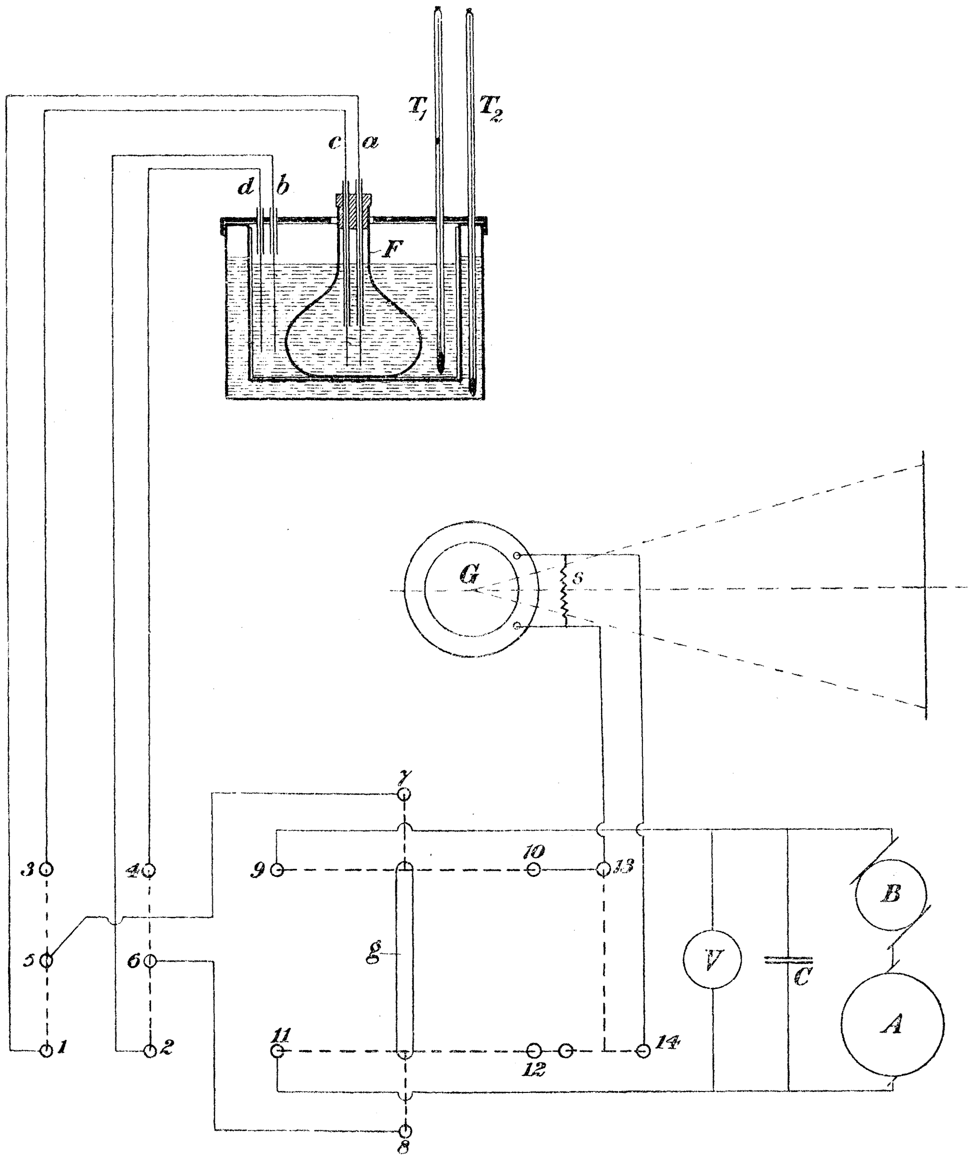

Fig 1

is placed. Thermometers, T_1 T_2, placed in the acid outside the jar and in the oil, are made to register the same, or nearly the same, temperatures when taking observations, but T_1 gives the temperature taken for the flask. The flask is heated by a Bunsen burner placed under the copper vessel. Two electrodes *a*, *c*, insulated from

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one another, and from the flask by means of sealing wax and glass tubes, dip into the sulphuric acid forming the inner coating of the jar, and similarly, electrodes b, d dipping into the outer acid make connection with the outer coating. The acid inside and out was made to wet the flask up to a level higher than the acid would reach at the highest temperatures.

The four electrodes, a, b, c, d, are connected respectively by thin copper wires, with four mercury cups 1, 2, 3, 4, cut in a block of paraffin, and, by means of a reversing switch a, b and c, d, can be connected respectively to mercury cups 5, 6. Cups 5, 6 are connected respectively to 7, 8 by thin wires, which can in turn be connected with or disconnected from the source of charge 9, 11.

The steady potential difference of about 1,500 volts is obtained from a Siemens alternator A, in series with a revolving contact maker B fixed to the alternator shaft and making contact once per complete period, there being six periods per revolution. The contact-maker is set to make contact when the potential difference is a maximum. A condenser C, and a Kelvin vertical electrostatic voltmeter V, are placed in parallel between the connecting wires leading to mercury cups 9, 11.

The galvanometer G has a resistance of 8,000 ohms and is inclosed in an iron box which acts as a magnetic shield. The box is supplied with a small window for the ray of light to pass through it from an incandescent lamp to the mirror from which it is reflected back through the window to a scale at a distance of 12 feet from the mirror. The divisions on this scale are $\frac{1}{10}$th of an inch apart, and an average sensibility for this instrument is $\cdot 3 \times 10^{-9}$ ampere per division of the scale. The galvanometer is supplied with a shunt S, and has its terminals connected to mercury cups 13, 14 on the paraffin block. These mercury cups are connected to cups 10, 12 respectively, which can at will be connected to 7, 8, by one motion of the glass distance-piece g forming part of the reversing switch which places 9, 11, or 10, 12, in contact with 7, 8. A switch is so arranged that 13, 14 can be connected at will, that is, the galvanometer is short circuited.

The process of charging, discharging, and observing, is as follows:—Near the observer is a clock beating seconds which can be distinctly heard by the observer. Initially, the cups 9, 11, are disconnected from 7, 8; but 5, 1, and 6, 2, are connected. At the given moment the reversing switch is put over connecting 7, 9, and 11, 8; the jar is then being charged through electrodes a, b. This goes on for the desired time, during which charging volts and zero of the instrument are noted. At the end of the time required for charge, the main reversing switch is put over connecting 7, 10, and 8, 12; next the subsidiary switch is put over connecting 3 to 5 and 4 to 6, and on opening the short-circuiting switch, the spot of light is deflected and allowed to take up its natural state of movement determined by residual charge, readings being taken at stated epochs after discharge is started. This whole operation, including an adjustment of the shunt when necessary, was so speedily accomplished that reliable readings could be taken 5 seconds after discharge is started. By using two electrodes, polarization of electrodes is avoided, and the gradually-diminishing current through the galvanometer is that due to residual charge. The conductivity of the jar is determined by removing the glass distance-piece *g*, connecting 7 to 9, 8 to 12, and 10 to 11, and noting the steady deflection on the galvanometer for a given charging potential difference.

In the ice experiment, the conductors from 3, 4, are used both for charging and discharging. The form of condenser used when dealing with ice and liquid dielectrics is shown in fig. 2. It consists of seven platinum plates, *a*, *b*, *c*, *d*, *e*, *f*, *g*, each measuring 2 inches by 3 inches, and of a thickness ·2 millim., separated from each other by a distance

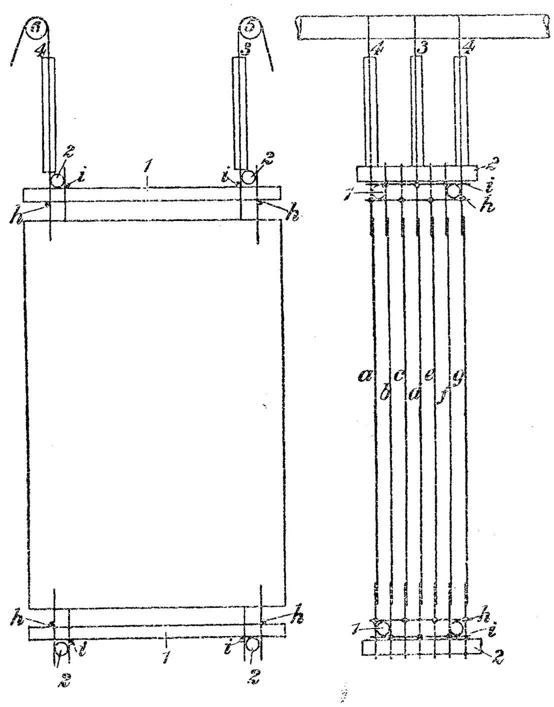


Fig 2.

of 2·7 millims. To each plate are gold-soldered four platinum wires—two top and two bottom. Plates *a*, *c*, *e*, *g*, form the outer coating of the condenser, and are kept in their relative positions by cross connecting wires *h*, gold-soldered to the wires at each end of each plate. Similarly, plates *b*, *d*, *f*, which form the other and inner coating of the condenser, are fixed relatively to one another by cross connecting wires *i*. The relative positions of the two sets of plates are fixed by glass rods 1, 2. The terminals of the condenser are, for the inner plates the prolonged wire 3, and for the outer plates the wires 4, 4. These are bent round glass rods 5, 6, which resting on the top of a beaker support the plates in the fluid. The glass tubes on the wires 3, 4, 4, are for the purpose of securing good surface insulation. The glass beaker is conical, so as to remain unbroken when freezing the distilled water within. This was accomplished by surrounding the beaker with a freezing mixture of ice and salt, the lower temperature being obtained by further cooling in carbonic acid snow.

The same blue flask, which was the subject of the earlier experiments, was mounted as shown in fig. 1, and the residual charge observed for various temperatures. This glass is composed of silica soda and lime; the colour is due to oxide of cobalt in small quantity.

Out of a large number of experiments the data in Tables I. and II. give the general character of the results.

TABLE I.

Time in seconds.	15° C.	34½.	54½.	70.	85.	117.	132.	Remarks.
10	246	..	..	..	..	9770	7256	Blue flask. 6th and 7th November, 1894. Sensibility of galvanometer, ·378 × 10⁻⁹. Duration of charge, 2 minutes. Charging volts, 1250
15	..	376	1176	2785	5445	..	..	
20	121	265	1030	2586	4100	3590	3010	
30	87	209	892	2070	2980	2150	1735	
60	46	131	683	1320	1510	950	778	
120	22½	91	483	720	688	440	350	
300	9¼	62	256	260	210	164	107	
600	..	..	123	110	..	86	less than 59	

TABLE II.

Time in seconds.	14° C.	55.	70.	110.	137.	Remarks.
10	205	..	..	11740	12400	Blue flask. 13th November, 1894. Sensibility of galvanometer, ·407 × 10^{-9}. Duration of charge, 2 minutes. Charging volts, 1250.
20	99	1230	2850	4790	4340	
60	38	837	1560	1212	990	
120	17	594	878	487	366	
300	5	308	314	134	..	

TABLE III.

	3rd January, 1895.			4 January, 1895.			Remarks.
Time in seconds.	8° C. 5 min. charge.	117 1 min.	117½ 1 min.	8° C. 5 min. charge.	122 1 min.	122½ 1 min.	
10	75	..	258	70	..	1100	New window glass flask. Sensibility of galvanometer, ·358 × 10^{-9}. Resistance of flask, 8° C., 3340 × 10^6 ohms. Charging volts, 1500.
20	43	123	148	44	..	664	
30	30	105	109	32	400	..	
60	18	80	73	19	240	240	
180	7½	..	56	7¾	69	68	

The figures given are the deflections of the galvanometer in scale divisions corrected for the shunt used. Recalling that one scale division means a known value in amperes, that a known potential in volts is used, these figures can readily be reduced to ohms $^{-1}$. The capacity of the flask is 0·0026 microfarad at ordinary temperatures and times, and the specific inductive capacity of its material under similar conditions is about 8. Hence one could reduce to absolute conductivities of the material. It is more interesting to consider how fast the capacity is changing. Take the first result given in Table III. for another flask 75 at 10 seconds; this means a conductivity $75 \times 0{\cdot}358 \times 10^{-9}/1{,}500 =$ about $0{\cdot}179 \times 10^{-10}$, and this is, of course, the rate in farads per second at which the capacity is changing in that experiment compared with a capacity of the flask $\frac{1}{2}10^{-3}$ microfarad measured with the shortest times, or, to put it shortly, the flask owing to residual charge is changing capacity at the rate of about 3 per cent. per second. These figures also show that the residual charge up to 20 seconds increases greatly with the temperature; the residual at 60 seconds rises with the temperature up to about 70° C. or 80° C., and then diminishes; residual charge at 300 seconds begins to diminish at about 60° C. One may further note the way in which the form of the function $\psi(\omega)$ changes as temperature rises. Compare in Table I. the values for 20 and 30 seconds, the ratios are :—

Temperature . . .	15	$34\frac{1}{2}$	$54\frac{1}{2}$	70	85	117	132,
Ratio	1·39	1·27	1·16	1·25	1·38	1·67	1·74.

In other words, if we expressed $\psi(\omega)$ in the form C/t^m, we should find m first diminishes as temperature rises to 54°, then increases as the temperature further rises. This has an important bearing upon the effect of residual charge on apparent capacity and resistance.

It will be noticed that the residual charge, for the same time, at high temperatures, is somewhat greater in Table II. than I. The results in Table I. were obtained on November 7th, 1894; those in Table II. on November 13th, 1894. There is no doubt but that heating this glass and submitting it to charge when heated, alters the character of the results in such manner as to increase residual charge for high temperatures. To test this more thoroughly, a new flask was blown out of window glass composed of silica lime and soda without colouring matter, and on January 3rd, 1895, was charged and discharged in the ordinary manner. After the results given in Table III. for January 3rd were obtained, the flask was charged for 21 minutes at 1,500 volts, the direction of charge being reversed after 10 minutes, the temperature of the flask being 133°. We see that on January 4th, Table III., the same effect is observed, namely an apparent increase in residual charge for the same time at high temperatures. This may probably be attributed to a change in the composition of the material by electrolysis.

Capacity.

(a.) *Low Frequency.*

Fig. 3 gives a diagram of connections, showing how the apparatus is arranged for the purpose of determining the capacity of poor insulators, such as window glass or ice, at varying temperatures. This is a bridge method, the flask F being placed in series with a condenser of known capacity K, and on the other side non-inductive resistances R_1, R_2. By means of keys k_1, k_2, the bridge can be connected to the poles of a Siemens alternator A; its potential difference is measured on a Kelvin multicellular voltmeter V. On the shaft of the alternator is fixed the revolving contact-maker B, which makes contact once in a period, and the epoch can be chosen.

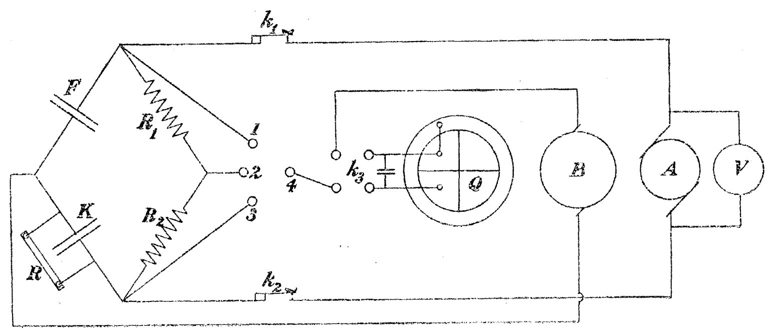


Fig 3.

The Kelvin quadrant electrometer Q has one pair of quadrants connected to a pole of the revolving contact-maker B, and the other to a mercury cup 4 in a block of paraffin. The other terminal of B is connected to the junction between F and K; by means of mercury cups 1, 2, 3, the electrometer can be connected through the contact-maker to either end, or to the middle of the bridge.

The compensating resistance R is the resistance due to pencil lines drawn on fine obscured glass strip,* about 12 inches long and $\frac{3}{4}$ inch wide, contact being made at each end by means of mercury in a small paraffin cup, and the whole varnished whilst hot with shellac varnish. A series of these resistances was made, ranging in value from a few megohms to a few tens of thousands of ohms. For the purpose of these experiments a knowledge of their actual resistance is of no moment, although for the purpose of manipulation their resistances are known.

The method of experiment is as follows:—Mercury cups 1 and 4 are connected by a wire, placing the electrometer and contact-maker across F, and the contact-maker is moved until it indicates no potential. Cups 3, 4 are now connected, and resistance R is adjusted until the electrometer again reads zero. After a few trials, alternately

* See 'Phil. Mag.,' March, 1879.

placing the bridge between 1, 4 and 3, 4, and adjusting R, the potentials are brought into the same phase, that is, the potential across the electrometer is zero in each case for the same position of the contact-maker. Mercury cups 2, 4 are now connected, the contact-maker B is adjusted to the point of maximum potential, and R_1, R_2 adjusted until balance is obtained. We now know that $K/F = R_1/R_2$.

k_3 is the ordinary key supplied with the electrometer, which reverses the charge on the quadrants or short circuits them. The range of frequency varies from 100 to 7 or 8 complete periods per second.

(B.) *High Frequency.*

For high frequencies a method of resonance is used,* and the apparatus shown in fig. 4. The primary coil consists of 1, 9, or 160 turns of copper wire 4 feet in diameter,

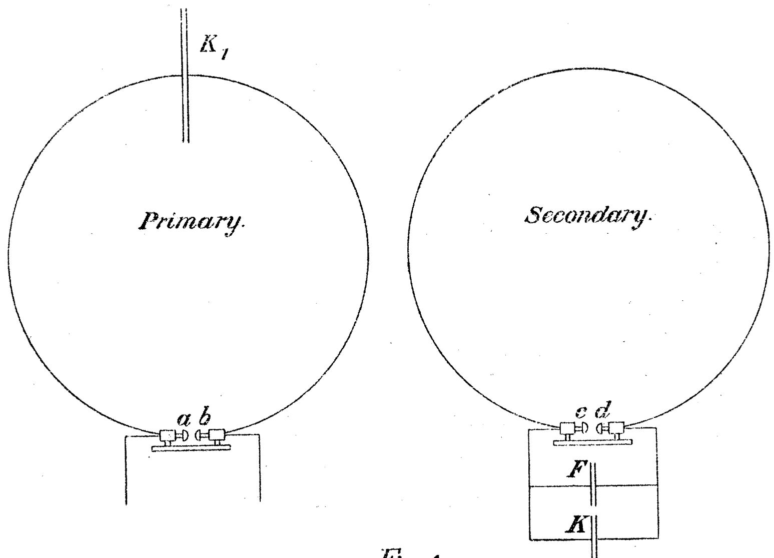


Fig 4.

having a condenser K_1 in its circuit and two adjustable sparking knobs *a*, *b*. The secondary is placed with its plane parallel to that of the primary, and usually at a distance of 4 or 5 feet from it; adjustable spark knobs *c*, *d* are provided in its circuit, which consists of 1, 9, or 160 turns of copper wire of the same diameter as the primary. The diameter of the wires for the 1, 9, and 160 turns are respectively 5·3, 2·65, and 1·25 millims. A Ruhmkorff coil excites the primary. Between the spark knobs *c*, *d* are placed the capacity to be found F, and a large slide condenser K. The method is one of substitution, that is to say, maximum resonance is obtained with both condensers attached by variation of K; F is removed and maximum resonance again obtained by increase of K. In order to bring K on the scale for the two maxima, it is necessary to adjust K_1, the condenser in the primary. This condenser consists of a

* This method, we find, has been used by THWING, 'Physical Society's Abstracts,' vol. 1, p. 79.

sheet of ebonite with tin-foil on either side; three such condensers are available, and by variation of the area of tin-foil, if necessary, a suitable value for K_1 was speedily obtained. Platinum-foil was used for the electrodes in the acid inside and outside the jar in the glass experiments instead of wire, as shown in fig. 1, in order to secure that the connections should not add materially to the self-induction of the circuit.

The frequency is calculated from the formula

$$\text{Frequency} = \frac{1}{2\pi\sqrt{\text{KL}\,10^{-15}}}$$

where

K is the capacity in secondary in microfarads,
L is the self-induction in centimetres.

$$\text{L} = 4\pi n^2 a \left(\log_\epsilon \frac{8a}{r} - 2\right)$$

where

n is the number of turns on the secondary,
$2a$ is the diameter of the ring = 4 feet,
$2r$ is the diameter of wire on secondary.

When $n = 1$, L = 4230 centimetres. If K be taken ·00096 microfarad the frequency is $2{\cdot}5 \times 10^6$.

The lowest frequency we have tried with this apparatus is when $n = 160$ $\text{L} = 136 \times 10^6$. If K be taken ·0028 microfarad, the frequency is 8,400.

That the capacity of some kinds of glass does not vary much with a moderate variation of temperature is known ('Phil. Trans.,' 1881, p. 365). Experiments were tried on the same blue flask as before, using the method in fig. 3. The results obtained and many times repeated for a frequency of 70 or 80 are given in Table IV. As the specific inductive capacity of this flask, measured in the ordinary manner, is about 8, it appears that at 170° it is about 21. Knowing from the results in Tables I. and II. how great was the residual charge for high temperatures and short times, it appeared probable that the result would depend upon the frequency. This was found to be the case, as shown by the results of November 26, 1894, Table IV., the apparent capacity being somewhat more than one-half at frequency 100 of what it is at frequency 7·3. Experiments on the window-glass flask show the same result.

The next step was to determine whether or not this large increase of apparent capacity was due to residual charge. To do this the resonance experiments fig. 4 were resorted to and the capacity of the flask was determined with a frequency of about 2×10^6; it was found to be sensibly the same whether the flask were hot or cold. The results show that the capacity varies from 185 to 198 in arbitrary units with a variation of temperature from $25\frac{1}{4}°$ to 127°. With frequency 8,400 the capacity varies from 240 to 285 in arbitrary units for a variation of temperature from 21° to 122°, but here the sensibility was not so good as with the higher frequency. We conclude that the apparently great capacity of this glass at a temperature from 120° to 170° is due to residual charge, but that the effects of this part of the residual charge are not greatly felt if the frequency is greater than about 10,000 a second.

The extent to which the capacity of the window-glass flask is affected by the frequency at ordinary temperatures, 8° C., is shown by the following figures :—

Frequency	12	39	70
Capacity	·00075	·0008	·001

TABLE IV.

20th November, 1894. Frequency, 72 ; volts, 70.		21st November, 1894. Frequency, $85\frac{1}{2}$; volts, $71\frac{1}{2}$.		26th November, 1894. Temperature, 120° C.		
Temperature. C.	Capacity of flask in terms of itself at 15°.	Temperature. C.	Capacity of flask in terms of itself at 25° C.	Frequency.	R_2/R_1.	Remarks.
15	1	$25\frac{1}{2}$	1	7·3	1·27	Standard condenser unaltered throughout experiment
92	1·31	54	1·05	12	1·11	
117	1·66	95	1·27	$39\frac{1}{2}$	·87	
154	2·6	120	1·59	$71\frac{1}{2}$	·78	
		170	2·61	100	·75	

CONDUCTIVITY AFTER ELECTRIFICATION FOR SHORT TIMES.

The Battery.—This consists of 12 series of small storage cells, fig. 5, each series containing 50 cells. The poles of each set of 50 cells are connected to mercury cups in a paraffin block, and numbered 1, 3, 5, . . . 21, 23, on the positive side; 2, 4, 6, . . . 22, 24, on the negative. Cups *b*, *d*, are connected to the poles of the 56 cells in the Laboratory, and therefore, by connecting *d*, 1, 2 . . . 21, 23, together on the one side, and 2, 4, . . . 22, 24, *b*, together on the other side, the cells can be charged in parallel. For the purpose of these experiments, a large potential difference is required; this is obtained by removing the charging bars, and replacing them by a series of conductors connecting *x* to 1, 2 to 3 . . . 22 to 23, 24 to *y*. In this manner, the whole of the 600 cells are placed in series with one another. Across the terminals *x*, *y*, are placed a condenser K_3 of about 4·3 microfarads, and a Kelvin vertical electrostatic voltmeter V. In order to change over quickly, and for the purpose of safety, the charging bars and connections for placing the cells in series are mounted on wood.

The Contact Apparatus.—This consists of a wooden pendulum carrying lead weights w_1, w_2, which were not moved during the experiments. The pendulum is released from the position p by the withdrawal of a brass plate, and, swinging forward, strikes a small steel contact piece f, carried on a pivoted arm of ebonite. The initial position of this ebonite arm is determined by a contact pin e, about $\frac{1}{16}$ inch diameter, contact being maintained by a spring m with an abutting rod insulated from a brass supporting tube by means of gutta-percha. This insulated rod is continued by a copper wire to the insulated pole of a quadrant electrometer Q. The brass supporting tube is continued by means of a metallic tape covering on the outside of the insulated wire, and is connected to the case and other quadrant of the electrometer. If, then, the pendulum be released from position p, the time which elapses between the terminal piece g first touching the plate f, and the time at which contact is broken between e and the insulated stop is the shortest time we have been able to employ in these experiments, its duration being ·00002 second.

For longer times an additional device, shown in plan only, is used. It consists of a brass pillar h, which carries a steel spring S, and which is moved to and fro in V-shaped slides by means of a screw provided with a milled head n, which is divided into twenty equal parts on the outside surface. A pointer fixed to the frame indicates the position of the head, and a scale on the brass slide shows the number of revolutions of the head from zero position. The pendulum steel piece g is of sufficient width to touch the spring S as it moves forward and strikes the plate f. The zero of the spring S is determined electrically by moving forward the pillar h, and noting the position of the milled head when contact is first made, the steel piece g being in contact with f, but not disturbing its initial position. The plate f is connected by a flexible wire with the slides which are in connection with the spring S through its support h. When, therefore, the spring S leads the plate f by any distance, the time of contact is that time which elapses between g first striking S and the severance of contact between the pin e and its stop, always supposing that g keeps in contact with S. A good deal of trouble was experienced before making this contact device satisfactory. The ebonite arm carrying e and f was originally of metal, f being insulated; but inductive action rendered the results untrustworthy. Then again, the spring S, when first struck by the pendulum evidently again severed contact before f was reached. To get over this difficulty a subsidiary series of fine steel wires were attached to S, so that as the pendulum moves forward the wires are one after the other struck. In order that the pendulum should not foul these wires or the spring S on its return to position p, it was slightly pressed forward by the hand at its central position.

The method adopted is that of the bridge. Starting from mercury cups x, we proceed by a fine wire to the terminal i, and thence, by a wire passing down the pendulum, to g. From g we pass through spring S and the piece f during contact to one end of the bridge. The flask F, or condenser to be experimented upon, is placed in series with metallic resistances a, these forming one arm of the bridge, the condensers K_1, K_2 forming the other arm. The stop e is connected to the junction between a and F; and the junction of K_1, K_2 is connected to the

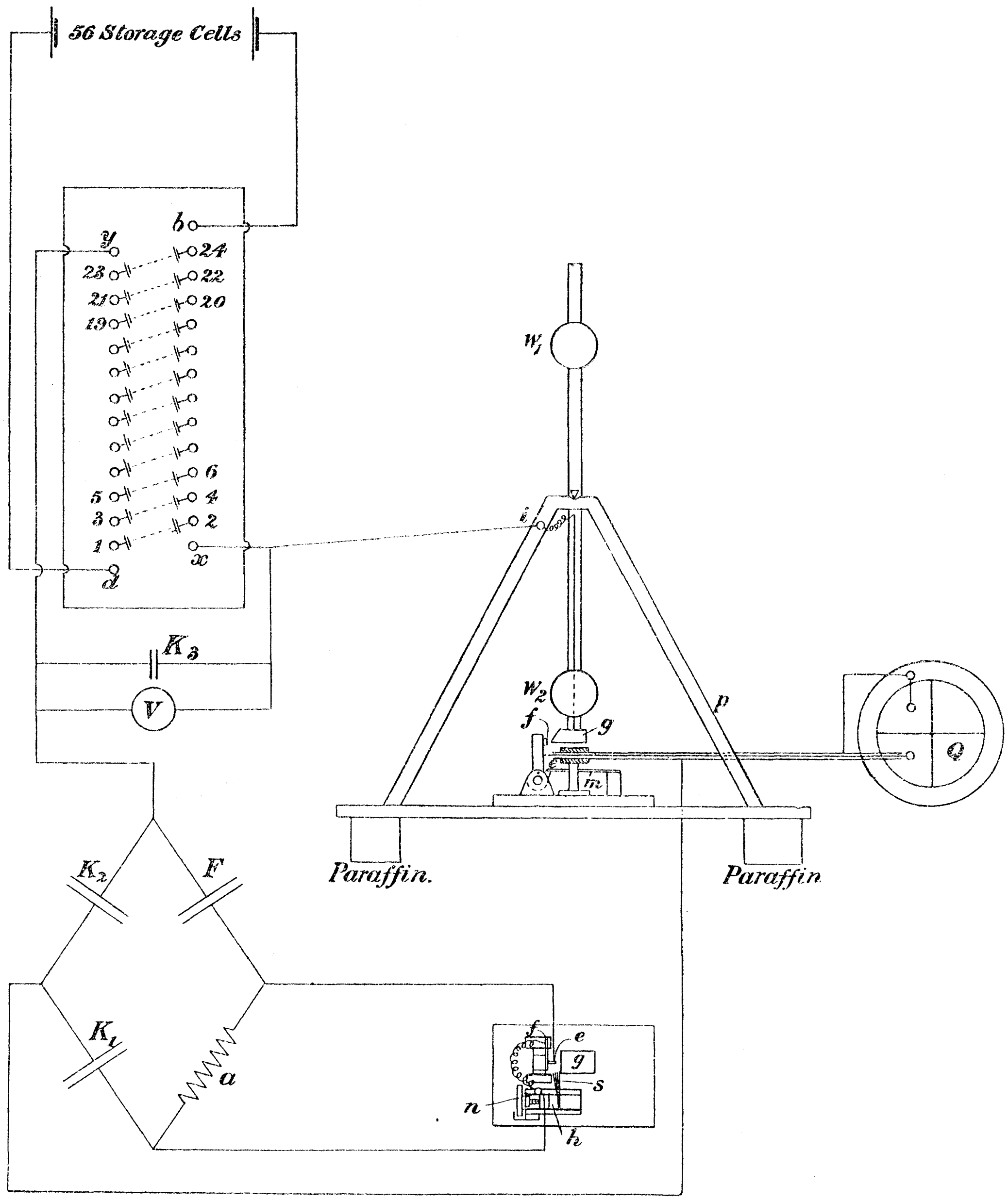


Fig 5

case of the electrometer by the outer conductor of the insulated wire leading to the instrument. The whole of the pendulum arrangement is supported on paraffin feet.

In the first instance pencil lines on glass were used for a, and K_1, K_2; but, for short times and varying current densities it was proved that these were

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

unreliable, when a knowledge of their actual resistance at the time of contact is taken to be the same as measured in the ordinary way on a Wheatstone bridge.

Time of Contact.—The connections were altered from those in fig. 5 to those in fig. 6. Eight dry cells having low internal resistance were used for charging. In fig. 6 let K be the capacity of the condenser, equal to $\frac{1}{3}$ microfarad. Let k be the capacity of the quadrant electrometer at rest in zero position, equal to ·000015 microfarad. Let R be the insulation resistance of K, and r the resistance through which the condensers are charged. Let E be the E.M.F. of the battery, V be the E.M.F. of condenser, and t the time of contact in seconds.

Then

$$(K + k)\dot{V} + \frac{V}{R} = \frac{1}{r}(E - V)$$

$$V = \frac{RE}{R + r}\left(1 - \epsilon^{-t\frac{R + r}{Rr} \cdot \frac{1}{K + k}}\right).$$

To determine E. Let $R = \infty$, $K = 0$, $r = 0$; the deflection of the electrometer needle from zero after the pendulum has struck gives E in scale divisions.

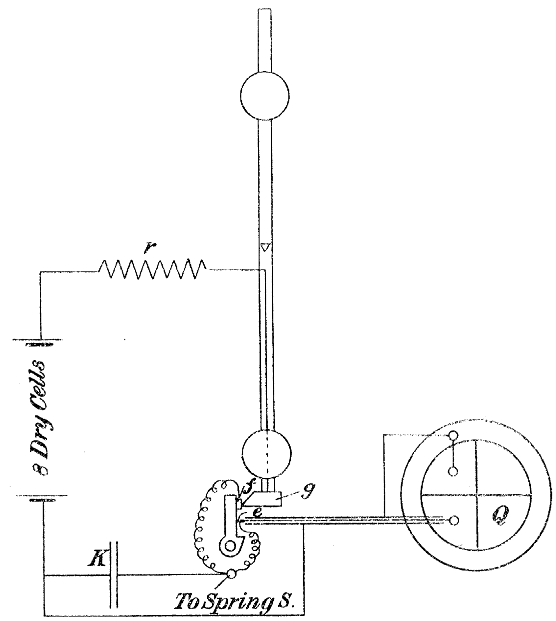


Fig 6.

To determine t. Let K be known and great as compared with k. Let $R = \infty$, and let r be such that the steady deflection from zero, V, after pendulum has struck, is about equal to half E.

$$V = E\left(1 - \epsilon^{-\frac{t}{Kr}}\right),$$

which gives

$$t = rK \log_{\epsilon} \frac{E}{E - V}.$$

The following are the values of t in seconds, so deduced, in terms of revolutions of the milled head n from zero :—

TURNS of milled head from zero.	0	$\frac{1}{2}$	1	2	3	4	5	6
TIME of contact in seconds.	·00002	·00035	·00099	·0028	·006	·009	·011	·014

The experiments have so far dealt with frequencies ranging from 2×10^6 to 8,000, and 100 to 10. The gap between 8,000 and 100, during which the great effects of residual charge become apparent, is filled up by experiments with the pendulum apparatus just described. An attempt was made to fill up this gap by means of the method shown in fig. 6, from which the effect on the capacity could be found for various times of contact, but this method was finally abandoned and used only for the determination of times of contact.

Referring to fig. 5, F is the same window-glass flask mentioned above, and mounted as in fig. 1 ; a is a non-inductive metal resistance, the effect of the capacity of which was at the most, when a is large, only capable of disturbing our experiments to the extent of eight per cent., but in most cases the disturbance is a small fraction of this ; K_1 is a one-third microfarad condenser, and K_2 the large slide condenser used in the other experiments. The advantage of this method of experiment is that the charging potential difference V is great, and the actual ohmic resistance of a is small as compared with that of the flask F. In this manner the effect of the instantaneous capacity of the flask is overcome at once and the after effects due only to residual charge can be examined directly. The results are shown in Table V.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

TABLE V.—Window-Glass Flask. 16th–31st October, 1896.

a. Resistance box.
K_1. $\frac{1}{3}$rd $m.f.$ = 118,000 divisions of large slide condenser.
K_2. Large slide. When at zero = 100 of its own scale divisions.
,, ,, When at 435 = ·00146 $m.f.$
K = ·0005 $m.f.$ from highest frequency resonance experiments.
In the diagram, fig. 7, giving curves of conductivity and time for given temperature,

$$1 \text{ centim. vertical} = 2 \times 10^{-8} \ (l^{-1} t).$$

$$1 \text{ centim. horizontal} = 2 \times 10^{-4} \ (t) \text{ seconds.}$$

Therefore, area × 4 × 10^{-6} gives capacity in microfarads.

Time of contact.		Temperature of Flask, 15·4° C.								
Turns of milled head.	Time in seconds.	Large slide position K_2	a in 10^6 ohms.	$\epsilon^{-\frac{t}{aK}} = A$.	Resistance of Flask c in 10^6 ohms. From $\frac{a + Ac}{a+c}$.	Resistance of Flask c in 10^6 ohms. From $a\frac{K_1}{K_2}$.	$\frac{1}{c}$ in 10^{-6} $(l^{-1} t)$.	$\int \frac{1}{c} dt - \frac{1}{c_\infty} t$. Area in square centims.	$\int \frac{1}{c} dt - \frac{1}{c_\infty} t$. $m.f.$	$F = K + \int \frac{1}{c} dt - \frac{1}{c_\infty} t$ in $m.f.$
0	·00002	160	·0056	·000794	3·99	2·5	·286			
0	·00002	430	·0090	·00117	2·80	2·0				
·5	·00035	..	..	..	..	..	..	11·0	·000044	·000544
1·0	·00099	160	·105	..	48	48	·0208	16·25	·000065	·000565
1·0	·00099									
2·0	·0028	70	·130	..	90	90	·0111	22·05	·000088	·000588
3·0	·0060									
	∞					1000 rough	·001			
		54–57·3° C.								
0	·00002	160	·00608	·00139	7·45	2·76	·200			
0	·00002	430	·0094	·00142	3·27	2·10				
·5	·00035	430	·055	1/330000	12·2	12·2	·085	10·7	·000043	·000543
·5	·00035	160	·025	$1/1·6 \times 10^{12}$	11·4	11·4				
1·0	·00099	160	·065	..	29·5	29·5	·035	18·6	·000074	·000574
1·0	·00099	160	·062	..	28·1	28·1				
2·0	·0028	70	·075	..	52	52	·019	26·2	·0001	·0006
3·0	·0060									
	∞					170	·0059			

TABLE V.—Window-Glass Flask. 16th–31st October, 1896—(continued).

Time of contact.		Temperature of Flask, 80° C.								
Turns of milled head.	Time in seconds.	Large slide position K_2.	a in 10^6 ohms.	$\epsilon^{-\frac{t}{aK}} = A$.	Resistance of Flask c in 10^6 ohms. From $\frac{a + Ac}{a + c}$.	From $a\frac{K_1}{K_2}$.	$\frac{1}{c}$ in 10^{-6} $(l^{-1}t)$.	$\int \frac{1}{c}\,dt - \frac{1}{c_\infty}\,t$. Area in square centims.	$m.f.$	$F = K + \int \frac{1}{c}\,dt - \frac{1}{c_\infty}\,t$ in $m.f.$
0	·00002	160	·0041	·000058	2·52	1·86	·435			
0	·00002	430	·0079	·0063	2·04	1·80				
·5	·00035	430	·023	$1/16 \times 10^{12}$	5·1	5·1	·196	23·3	·000093	·000593
1·0	·00099	430	·066	..	14·7	14·7	·068	38·25	·000153	·000653
2·0	·0028	430	·100	..	22·3	22·3	·045	54·1	·000216	·000716
3·0	·0060	160	·065	..	29·5	29·5	·034	76·3	·000381	·000881
	∞					65	·0154			
		110–112° C.								
0	·00002	430	·0047	·00020	1·88	1·05	·714			
0	·00002	430	·0043	·000091	·973	·96				
·5	·00035	430	·0177	$1/16 \times 10^{16}$	3·9	3·9	·256	24·5	·000098	·000598
1·0	·00099	430	·020	..	4·5	4·5	·222	44·1	·000176	·000676
2·0	·0028	430	·025	..	5·6	5·6	·178	78·7	·000315	·000815
3·0	·0060	430	·029	..	6·5	6·5	·154	113·6	·000568	·00107
	∞					8·5	·118			

Window-Glass Flask. 2nd November, 1896.

Time of contact.		Temperature of Flask, 126° C.								
0	·00002	430	·0035	1/89000	·779	·780	1·28			
·5	·00035	430	·0082	..	1·83	1·83	·546	32·7	·000131	·000631
1	·00099	430	·0105	..	2·34	2·34	·427	55·5	·000222	·000722
2	·0028	430	·012	.	2·68	2·68	·373	70·8	·000283	·000783
3	·0060	430	·0133	..	2·97	2·97	·337	102·2	·000511	·000101
4	·009	430	·0135	..	3·0	3·0	·333	103	·000515	·00101
5	·011	430	·0135	..	3·0	3·0	·333	103	·000515	·00101
	∞					3·00	·333			
		145° C.								
0	·00002	430	·0018	$1/4·36 \times 10^9$	·401	·401	2·49			
·5	·00035	430	·0032	..	·713	·713	1·40	50·8	·000255	·00075
1	·00099	430	·004	..	·892	·892	1·12	83	·00033	·00083
2	·0028	430	·0043	..	·959	·959	1·04	93	·00037	·00087
3	·0060	430	·0043	..	·959	·959	1·04	93	·00037	·00087
4	·009									
5	·011									

TABLE V. (continued).—10th November, 1896.

Window-Glass Flask. Solder used instead of Acid.

Time of contact.	a in ohms.	Resistance of flask c in 10^6 ohms.	Temperature of flask °C.	$\frac{1}{c}$ in 10^{-6} ohms^{-1}.
⎧ ·00002	·7	·000156	about 350	6410
⎨ ·00099	1·5	·000334	" 350	3000
⎨ ·0028	1·5	·000334	" 350	3000
⎩ ·011	1·5	·000334	" 350	3000
·00002	18	·00446	285	224·0
·00002	130	·0290	229	34·5
·00002	200	·0446	219	22·4
⎧ ·00002	270	·0602	203	16·6
⎨ ·00099	330	·0736	202	13·6
⎨ ·006	370	·0825	200	12·1
⎩ ·014	370	·0825	200	12·1
Summary of results with acid.				
·00002	1000	·223	160	4·48
·00002	1800	·400	143	2·5
·00002	4700	1·05	112	·71
·00002	4100	1·86	80	·43
·00002	6080	2·76	55	·2
·00002	5600	3·99	15	·28

Let K_1, K_2, and F be discharged and let the potential difference V be applied to the bridge for time t. Let c be the ohmic resistance of the flask at the end of time t. Let K be its instantaneous capacity which is found by resonance at frequency 2×10^6. Let v be the potential across a. Then

$$\frac{v}{a} = \frac{V - v}{c} + (V - v)^{\cdot}$$

$$v = \frac{Va}{a + c}\left\{1 + \frac{c}{a}\,\epsilon^{-\frac{a+c}{acK}t}\right\}.$$

We know a, K, and t, and measure $\frac{a}{a+c}\left\{1 + \frac{c}{a}\,\epsilon^{-\frac{a+c}{acK}t}\right\}$.

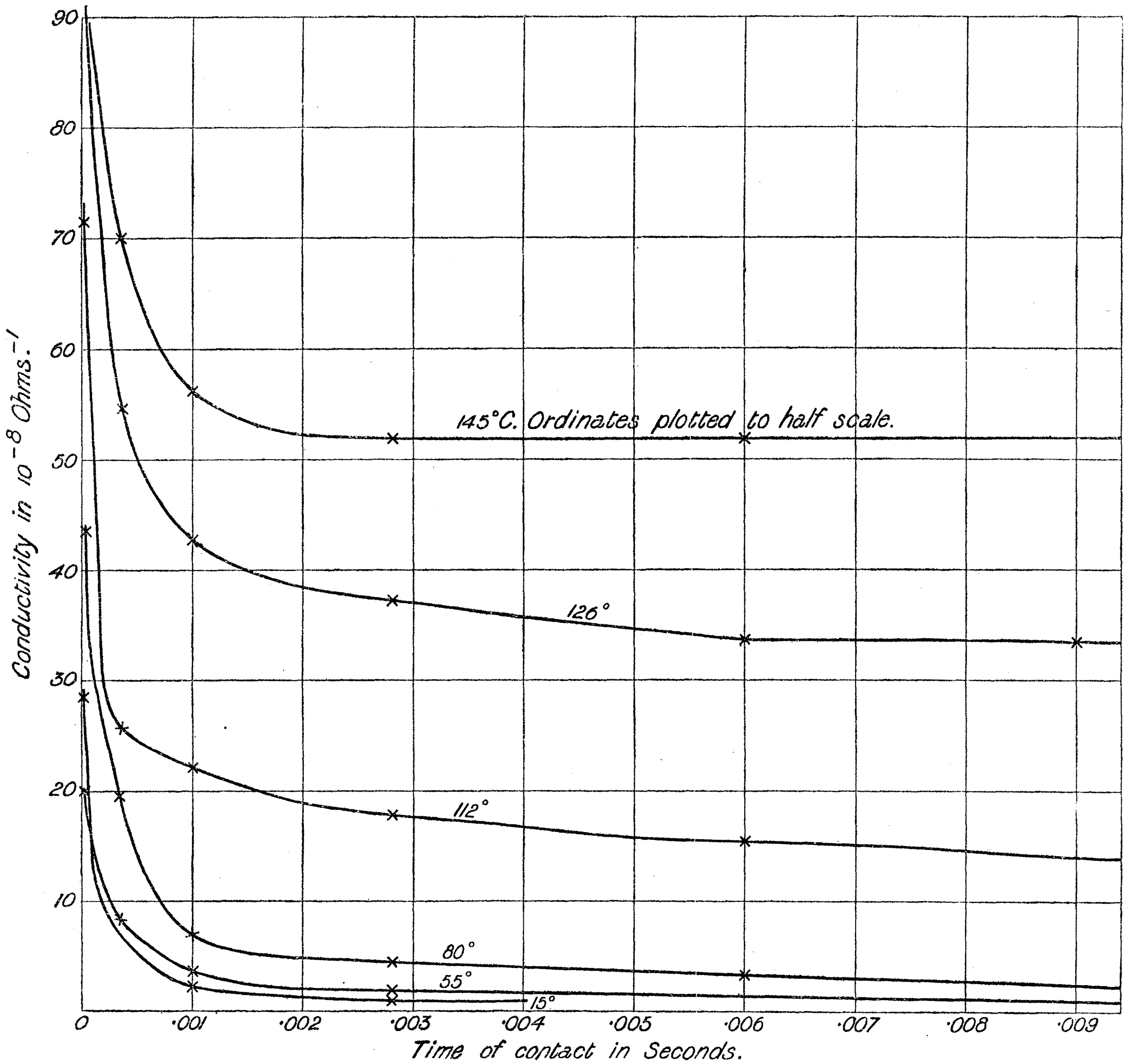
Now c is large compared to a, hence $\frac{a+c}{ac} = \frac{1}{a}$, therefore $\epsilon^{-\frac{a+c}{acK}t}$ is known; let it equal A. Then we have

$$\frac{a + Ac}{a + c} = \frac{K_2}{K_1 + K_2}.$$

We have reduced a consistent with fair sensibility until the correction due to instantaneous capacity is so small as to be almost negligible, that is, until $\frac{a + \mathrm{A}c}{a + c}$ is sensibly equal to $\frac{\mathrm{K}_1}{\mathrm{K}_2 + \mathrm{K}_1}$.

How far we have been able to carry this can be seen by an inspection of Table V. It is only for the shortest time of contact that the correction for $\epsilon^{-\frac{t}{a\mathrm{K}}}$ becomes at all sensible.

Fig. 7.



All temperatures from 15° to 145° were obtained by heating the flask as mounted in fig. 1; for 200° to about 350° acid was taken away and a solder, melting at about 180° C., substituted. Since the solder only half filled the flask the conductivity should be about doubled for 200° to 350° when comparing with the lower temperatures.

Since $\frac{1}{c_t}$ is the conductivity of the jar at time t, let curves of conductivities be drawn in terms of times of contact in seconds. Fig. 7 gives these curves, which have been plotted from Table V. They show that, after a given time of contact, the effect of residual charge gradually diminishes as the temperature increases, until only the conductivity of the jar for infinite times is experienced. For instance, at about a temperature of 250° the table shows that the whole effect of residual charge has died away after 1/10,000 of a second. The total capacity of the jar at time t will be $K + \int_0^t \frac{1}{c}\, dt - \frac{1}{c \infty} t$; where K is the instantaneous capacity which has been found by resonance to be = ·0005 microfarad for frequency 2×10^6.

K_1 = 118,000 divisions of the large slide condenser.

The curves in fig. 7 have been integrated, and their area up to ·0028 second, when reduced to microfarads and added to K, shows that, for time of contact ·0028 second, the total capacity, which is ·000588 at temperature 15·4°, is ·00087 at temperature 145°. This total capacity diminishes as the times of contact diminish, until we get to the results which resonance has shown; and then the capacity of this flask is sensibly the same for all temperatures when the frequency is of the order 2×10^6 per second.

ICE.

Ice was next examined, both in regard to its residual charge and its capacity. The residual charge is considerable, and increases as the temperature rises. Table VI. gives the residual charge of ice at two temperatures: the higher is produced by a freezing mixture of ice and salt, and is about − 18° C.; the lower by placing carbonic acid snow round the beaker, the whole being wrapped in thick felt. The apparent capacity depends on the frequency, as shown by the results in Table VII. At − 18° the capacity is twice as great with frequency 10 as with 77·6. At the lower temperature the capacity is greater for frequency 9 than for frequency 77·6, in the ratio 1·39 to unity.

The specific inductive capacity of ice was next determined, with a high frequency, by resonance: it was found to be about 3.* Decreasing the frequency to about 10,000 rendered the method by resonance less sensitive, but it is certain that the specific inductive capacity is, for this frequency, of the order 3 rather than 50. We conclude that the great deviation of ice from MAXWELL'S law is due to residual charge, which comes out between frequencies 10,000 and 100.

Our next step was to determine the resistance c, as in the case of glass, by the method shown in fig. 5. The platinum plates, fig. 2, were used, and to observe the temperature of the ice a platinum wire of resistance 1·32 ohms at 0° C. was frozen in the ice and surrounded the condenser. Table VIII. gives the results. K the capacity as given by the resonance experiments with frequency 2×10^6 was ·00022 microfarad. Adding to this $\int_0^t \frac{1}{c}\, dt - \frac{1}{c_\infty} t$, we find that at time ·0028 the total

* THWING finds 2·85 to 3·36; BLONDLOT 2; PERROT 2·04.

capacity is ·0038 at $-30°$ C., whereas it is for the same time ·0065 at $-18°$ C. The curves of conductivity are given in fig. 8, and show the same character of results as those in the case of glass, fig. 7.

Fig. 8.

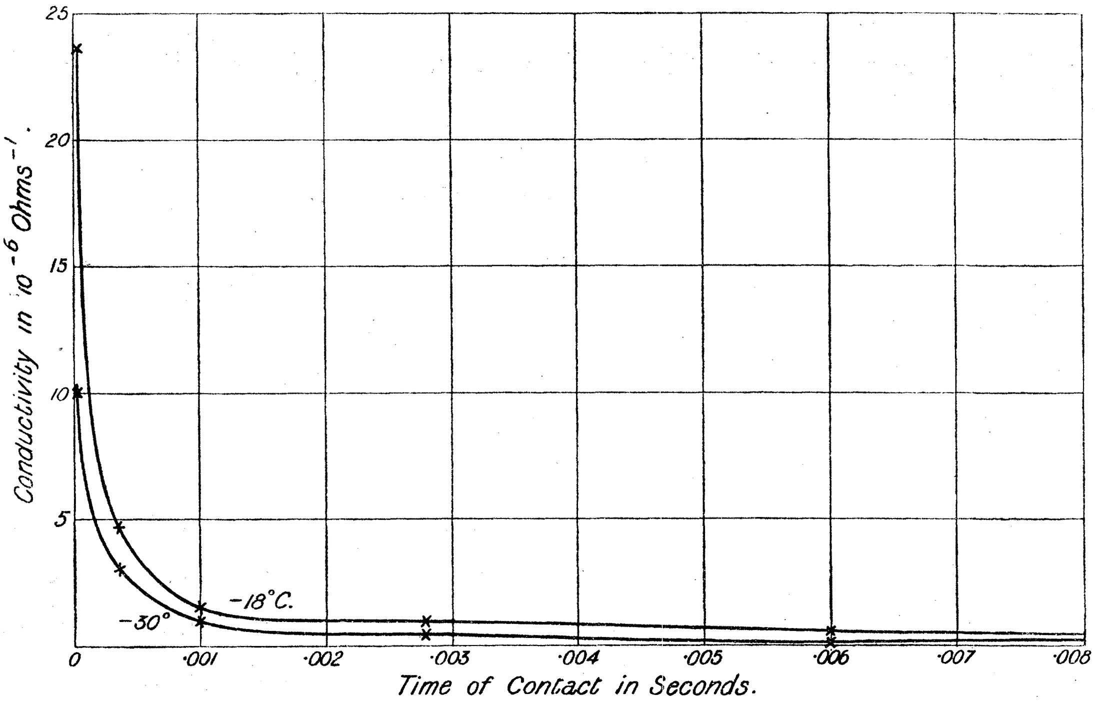


TABLE VI.

Time in seconds.	About − 18° C.	About − 30°.	Remarks.
10 20 60 90	2800 760 377 347	866 314 74 44	Charging volts 890. 8th December, 1894. Duration of charge, ¼ minute in each case. Resistance at 945 volts. − 18° C., 7·2 × 10^6 ohms; − 30°, 32·5 × 10^6 ohms.

TABLE VII.

8th December, 1894, − 18° C. about.		8th December, 1894, − 30° C. about.	
Frequency.	Capacity.	Frequency.	Capacity.
77·6 10	·01 ·019	77·6 9	·0072 ·01

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Table VIII.—Ice. 5th November, 1896.

a = Resistance box.
K_1 = $\frac{1}{3}$rd *m.f.*
K_2 = Large slide condenser.
K = Instantaneous capacity of ice condenser = ·00022 *m.f.* See Resonance, 19th November, 1895.

Time of contact.		Temperature of Ice, – 18° C.								
Turns of milled head.	Time in seconds.	Large slide position K_2.	a in 10^6 ohms.	$\epsilon^{-\frac{t}{a\mathrm{K}}} = \mathrm{A}$.	Resistance of Condenser c in 10^6 ohms. From $\frac{a + \mathrm{A}c}{a + c}$.	Resistance of Condenser c in 10^6 ohms. From $a\frac{\mathrm{K}_1}{\mathrm{K}_2}$.	$\frac{1}{c}$ in 10^{-6}	$\int \frac{1}{c}\,dt - \int \frac{1}{c_\infty}\,dt$. Area in square centims.	$\int \frac{1}{c}\,dt - \int \frac{1}{c_\infty}\,dt$. *m.f.*	$\mathrm{K} + \int \frac{1}{c}dt - \frac{1}{c_\infty}t$ in *m.f.*
0	·00002	430	·00019	10^{-200}	·0424	·0424	23·6			
			·00022							
$\frac{1}{2}$	·00035	430	·00096	. .	·214	·214	4·67	16·8	·0034	·00362
			·0010							
1	·00099	430	·003	. .	·669	·669	1·49	25·15	·005	·00525
			·003							
2	·0028	430	·0059	. .	1·32	1·32	·758	31·4	·00628	·00650
			·0060							
3	·006	430	·0078	. .	1·74	1·74	·575	37·4	·0075	00772
			·0079							
4	·009	430	·0095	. .	2·12	2·12	·472	39·2	·0078	·00802
5	·011	430	·011	. .	2·45	2·45	·408	40	·008	·00822
			·011	. .	*2·45*	*2·45*				
6	·014	. .	·011	. .	2·45	2·45	·408	40	·008	·00822
7	. .	. .	·012	. .	2·68	2·68	·373	40	·008	·00822
			·013	. .	*2·90*	*2·90*	*·345*			
9	. .	*430*	*·013*	. .	2·90	*2·90*	*·345*	40	·008	·00822
The italics give the result of a second experiment.										
		Temperature of Ice, – 32° C to – 27° C.								
0	·00002	430	·00045	$1/7 \times 10^{-86}$	·100	·100	10			
$\frac{1}{2}$	·00035	430	·0015	. .	·334	·334	2·99	9·3	·00186	·00208
1	·00099	430	·005	. .	1·12	1·12	·893	14·2	·00284	·00306
2	·0028	430	·014	. .	3·12	3·12	·320	17·9	·00358	·00380
3	·006	430	·021	. .	4·68	4·68	·214	22·4	·0045	·0047
4	·009	430	·0255	. .	5·69	5·69	·176	23·9	·0048	·0050
5	·011	430								
6	·014	. .	·032	. .	7·13	7·13	·140	25·1	·0050	·0052
8	. .	430	·039	. .	8·7	8·7	·115			
10	. .	. .	·043	. .	9·59	9·59	·104			
11	. .	430	·045	. .	10·0	10· 0	·100			

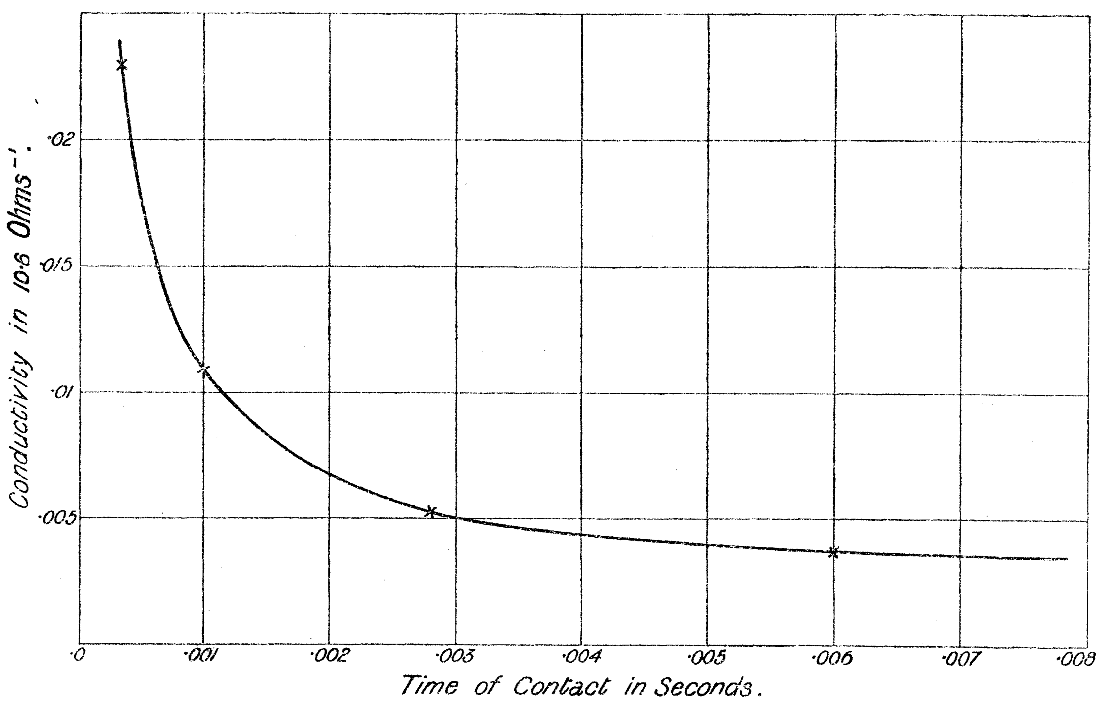
[Added January 18th, 1897.]

Castor Oil.

This oil was obtained from Messrs. Hopkin and Williams, and was tested as supplied. The platinum plates, fig. 2, were submerged in this oil. Resonance experiments give, for frequency 2×10^6, a capacity equal to 105 divisions on the large slide condenser. For long times the method was not that shown in fig. 3, but a bridge method, used in the earlier experiments,* in which a Ruhmkorff coil is used for exciting. This test gives 139 scale divisions on the same slide condenser. In air the plates have capacity 30 scale divisions. We see, therefore, that at frequency 2×10^6 the specific inductive capacity would be 3·5 as against 4·63 for long times.

The short-time contact experiments, fig. 5, give the results in Table IX., the temperature of the oil being 6°C, from which we see that residual charge in this oil is considerable. The total capacity after time of contact ·006 second is ·00034; whereas, with high frequency by resonance, it is ·000287 microfarad. The curve in fig. 9 gives the relation between conductivity and time of contact, and has been plotted from Table IX.

Fig. 9.



* See 'Proc. Roy. Soc.', vol. 43, p. 156.

TABLE IX.—Castor Oil. 16th November, 1896.

$K_1 = \frac{1}{3}$ microfarad Condenser; K_2 = large slide Condenser.
K = ·000287 microfarad from High Frequency Resonance Experiment.

Time of contact.		Temperature of Castor Oil, 6° C.								
Turns of milled head.	Time in seconds.	Large slide position K_2.	a in 10^6 ohms.	$\epsilon^{-\frac{t}{aK}} = A$.	Resistance of Flask c in 10^6 ohms. From $\frac{a + Ac}{a + c}$.	From $a\frac{K_1}{K_2}$.	$\frac{1}{c}$ in 10^{-6} $(l^{-1} t)$.	$\int \frac{1}{c} dt - \frac{1}{c_\infty} t$. Area in square centims.	$m.f.$	$F = K + \int \frac{1}{c} dt - \frac{1}{c_\infty} t$ in $m.f.$
0	·00002	430	·015	1/100	− 2·7	3·34	·3*			
½	·00035	160	·095	1/361000	43·1	43·1	·023	6·2	·000025	·00031
1	·00099	40	·110	..	..	92·7	·0108	8·5	·000034	·00032
2	·0028	40	·223	..	..	188	·0053	12	·000048	·00033
3	·006	40	·320	..	..	270	·0037	14	·000056	·00034
5	·01	40	·460	..	..	388	·00258			
7	..	40	·620	..	..	523	·0019			
9	..	40	slightly greater than ·620							

GLYCERINE.

This glycerine was obtained from Messrs. HOPKIN and WILLIAMS, and has been tested for purity and dried very carefully by Mr. HERBERT JACKSON, of the Chemical Department of King's College, London. The platinum plates, after careful cleaning in benzene, caustic-potash, and water were thoroughly dried and submerged in the glycerine in a beaker, the whole being placed in a glass receiver over a strong dehydrating agent. After exhaustion, just sufficient air was admitted to render the space inside sufficiently non-conducting to stop discharge between the terminals of the condenser which are sealed into glass tubes supported by an indiarubber stopper. The short-contact experiments show that the apparent resistance is 60,000 ohms, whether the time of contact be ·00002 or ·001 second, showing that there is no residual charge. The resonance experiments with high frequency give ·005 microfarad for the capacity with glycerine, whereas with air the condenser had ·000082 capacity; the specific inductive capacity is, therefore, about 60. A test made as with castor oil with a Ruhmkorff at low frequency was difficult, but a fair approximation

* Taken from $a\frac{K_1}{K_2}$, since the negative value obtained from $\frac{a + Ac}{a + c}$ is untrustworthy, probably owing to K being still too large. To satisfy MAXWELL's law, K should = ·000176 microfarad.

was made by introducing a suitable compensating leakage into one of the other condensers of the bridge.* The result indicated a capacity between 50 and 60.

Water.

The platinum plates (fig. 2) were placed in ordinary distilled water in a beaker which was cooled to 0° C. by a surrounding brine solution composed of water, common salt and ice. The experiments with the short-contact apparatus show no material difference in the apparent resistance, whether the time of contact be ·00002 or ·00099 second; the apparent resistance for these times is 379 ohms. The effects of residual charge in water do not affect the resistance within the range of times of contact given by this apparatus.

[Added March 17th, 1897.]

Oil of Lavender.

This oil was supplied by Messrs. Hopkin and Williams: it was tested with the short-contact apparatus, fig. 5, $K_1 = ·33$, $K_2 = ·0015$ microfarad. The charging potential was 1250 volts; the following figures give the results:—

Time of contact in seconds . .	·00002	·00099	·0028	·006	·01
a in ohms	9500	14000	14500	14800	14800

The high frequency resonance experiments give specific capacity 3·89: the frequency being of the order 2×10^6.

Two experiments were made at low frequency. First, the Bridge method, fig. 3, which gives the following results, the temperature of the oil being 16°C. :—

Frequency	Charging Volts.	Specific Capacity.
18	65	5·6
79	30	4·34

Second, the Bridge method with Ruhmkorff coil as used in the castor oil experiments. Temperature 14°C. Specific capacity 4·18.

Experiments have been made by Stankewitsch ('Wied. Ann.,' 52), showing a variable capacity for oil of lavender. We, however, have not succeeded in obtaining any result so high as his.

* This appears to have been done by Nernst, 'Physical Society's Abstracts,' vol. 1, p. 38.